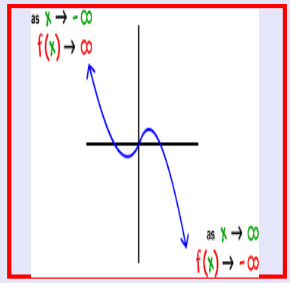
Math 245

Spring 2022

Lecture 32



Show that -3 is a solution of $x^4 - 13x^2 + 36 = 0$ ($0x^3$, $0x$)

-3 is a solution if synthetic division by -3 has a zero remainder.

Missing Terms

-3	1	0	-13	0	36
		-3	9	12	-36
	1	-3	-4	12	0

$0 \leftarrow$ Remainder $= 0$

This also tells us that $x-(-3)$ is a factor of $x^4 - 13x^2 + 36$.

Another word

$$x^4 - 13x^2 + 36 = (x+3)(x^3 - 3x^2 - 4x + 12)$$

Show $x-5$ is a factor of $x^3-5x^2+3x-15$.

$x-5$ is a factor only if remainder $=0$

$x-k$ $\qquad\qquad$ $f(5)=0$

We can show this by doing synthetic division by 5.

$$\begin{array}{r|rrrr} 5 & 1 & -5 & 3 & -15 \\ & & 5 & 0 & 15 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

$$x^3-5x^2+3x-15=(x-5)(x^2+3)$$

Find $P(x)=ax^2+bx+c$ with solutions $\frac{2}{3}$ and -4.

$x=\frac{2}{3}$, $x=-4$

$3x=2$,

$3x-2=0$, $x+4=0$

Factors of $P(x)$

So $P(x)=(3x-2)(x+4)$

Foil & Simplify

$$\boxed{P(x)=3x^2+10x-8}$$

Let's do synthetic division by $\frac{2}{3}$.

$$\begin{array}{r|rrr} \frac{2}{3} & 3 & 10 & -8 \\ & & 2 & 8 \\ \hline & 3 & 12 & 0 \end{array}$$

$\frac{2}{3}\cdot 3=2$

$\frac{2}{3}\cdot 12=8$

Remainder

What about -4?

$$\begin{array}{r|rrr} -4 & 3 & 10 & -8 \\ & & -12 & 8 \\ \hline & 3 & -2 & 0 \end{array}$$

Find polynomial $P(x) = ax^2 + bx + c$ with Solutions $\pm 5i$.

$x = 5i$, $x = -5i$

$x - 5i = 0$ $\quad$ $x + 5i = 0$

factors

$P(x) = (x - 5i)(x + 5i)$ → conjugates

$(A-B)(A+B) = A^2 - B^2$

$= x^2 - (5i)^2 = x^2 - 25i^2 = x^2 - 25(-1) = \boxed{x^2 + 25}$

Let's do synthetic division by $-5i$.

$$\begin{array}{r|rrr} -5i & 1 & 0 & 25 \\ & & -5i & -25 \\ \hline & 1 & -5i & 0 \end{array}$$

Remainder

Find a third degree polynomial $P(x)$ in the form of $P(x) = ax^3 + bx^2 + cx + d$ with Solutions 4, −3, and 5.

Zeros

$x = 4$ $\quad$ $x = -3$ $\quad$ $x = 5$ $\quad$ Solutions

$x - 4 = 0$ $\quad$ $x + 3 = 0$ $\quad$ $x - 5 = 0$ $\quad$ Make RHS=0

Factors of $P(x)$

$P(x) = (x-4)(x+3)(x-5)$

$= (x-4)(x^2 - 2x - 15)$

$= x^3 - 2x^2 - 15x - 4x^2 + 8x + 60$

$\boxed{P(x) = x^3 - 6x^2 - 7x + 60}$

To check, do synthetic division by −3.

$$\begin{array}{r|rrrr} -3 & 1 & -6 & -7 & 60 \\ & & -3 & 27 & -60 \\ \hline & 1 & -9 & 20 & 0 \checkmark \end{array}$$

Now do Synthetic division by 5

$$\begin{array}{r|rrr} 5 & 1 & -9 & 20 \\ & & 5 & -20 \\ \hline & 1 & -4 & 0 \checkmark \end{array}$$

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Find $P(x)=ax^3+bx^2+cx+d$ with solutions $\pm 3i$, and $\frac{1}{2}$.

$x=3i$ $\qquad x=-3i$ $\qquad x=\frac{1}{2}$ $\qquad$ Solutions

Make RHS=0

$x-3i=0$ $\qquad x+3i=0$ $\qquad x-\frac{1}{2}=0$

$2x-1=0$

Factors of $P(x)$

$P(x)=(x-3i)(x+3i)(2x-1)$ $\qquad \to P(x)$

$=(x^2+9)(2x-1)=\boxed{2x^3-x^2+18x-9}$

do synthetic division by one of solutions to verify that remainder is 0.

$$\begin{array}{r|rrrr} 3i & 2 & -1 & 18 & -9 \\ & & 6i & -18-3i & 9 \\ \hline & 2 & -1+6i & -3i & 0\checkmark \end{array}$$

Remainder

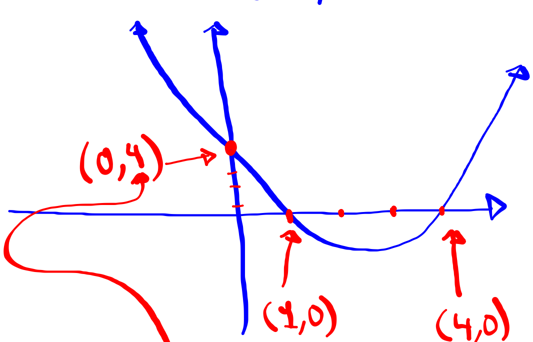
$3i(-1+6i)=$
$-3i+18i^2=$
$-3i+18(-1)=$
$-18-3i$

$3i(-3i)=$
$-9i^2=$
$-9(-1)=$
9

Check $\frac{1}{2}$

$$\begin{array}{r|rrr} \frac{1}{2} & 2 & -1+6i & -3i \\ & & 1 & 3i \\ \hline & 2 & 6i & 0\checkmark \end{array}$$

Consider the graph below



Y-Int (0,4)

X-Ints (1,0), (4,0)

$P(x)=ax^2+bx+c$

Solutions are X-Ints.

Solutions are 1 & 4.

$x=1$ $\qquad x=4$

$x-1=0$ $\qquad x-4=0$ $\qquad$ Factors=0

$P(x)=(x-1)(x-4) \Rightarrow \boxed{P(x)=x^2-5x+4}$

Find $P(x)=ax^2+bx+c$ with Solutions $2\pm3i$.

$x=2+3i$ $\qquad$ $x=2-3i$ $\qquad$ Solutions

Zeros

$x-2-3i=0$ $\qquad$ $x-2+3i=0$ $\qquad$ Factors $=0$

$P(x)=$ Product of all Factors

$=(x-2-3i)(x-2+3i)$

Conjugates $\rightarrow (A-B)(A+B)$

$=A^2-B^2$

$=(x-2)^2-(3i)^2$

$=x^2-4x+4-9i^2$

$=x^2-4x+4-9(-1)=x^2-4x+4+9$

$\boxed{P(x)=x^2-4x+13}$

SG 12

When dividing by synthetic division, it has to be in the form of $x-k$.

Coef. $=1$

Use synthetic division to show $2x-1$ is a factor of $P(x)=4x^2-1$.

$2x-1=2(x-\frac{1}{2})$

k

$$\begin{array}{c|ccc} \frac{1}{2} & 4 & 0 & -1 \\ & & 2 & 1 \\ \hline & 4 & 2 & 0 \end{array}$$

$P(x)=2(x-\frac{1}{2})(4x+2)$

$P(x)=(2x-1)(4x+2)$

$\neq 4x^2-1$

Discuss this tomorrow.